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# NLO-QCD corrections to $e^+e^- \to$ hadrons in models of TeV-scale gravity

**Prakash Mathews**

*Theory Group, Saha Institute of Nuclear Physics*
*1/AF Bidhan Nagar, Kolkata 700 064, India*
*E-mail:* prakash@theory.saha.ernet.in

**Vajravelu Ravindran**

*Harish-Chandra Research Institute*
*Chhatnag Road, Jhunsi, Allahabad, India*
*E-mail:* ravindra@mri.ernet.in

**Krishnamoorthy Sridhar**

*Department of Theoretical Physics, Tata Institute of Fundamental Research*
*Homi Bhabha Road, Mumbai 400 005, India*
*E-mail:* sridhar@theory.tifr.res.in

Abstract: We present results on NLO-QCD corrections to the process $e^+e^- \to$ hadrons via photon-, $Z$- and graviton-exchange in the context of TeV-scale gravity models. The quantitative impact of these QCD corrections for searches of extra dimensions at a Linear Collider is briefly discussed.

Keywords: Extra Large Dimensions, QCD, NLO Computations.

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Models of extra dimensions have dominated the recent theoretical literature on physics beyond the standard model. These models are attractive because the geometry of the compact extra spatial dimensions helps address the hierarchy between the electroweak and the Planck scales. In the model of Arkani-Hamed, Dimopoulos and Dvali (the ADD model) [1] a large magnitude of $d$ extra dimensions is responsible for lowering the Planck scale down to a TeV and so that one reduces the hierarchy between the Planck scale and the electroweak scale but at the expense of producing a much smaller hierarchy between the Planck scale and the size of the extra dimensions.

In the ADD model, the Standard Model (SM) fields are constrained to a 3-brane, while gravitons propagate in the $4+d$ dimensions. Then the size of the extra dimensions is only constrained by the length scales to which the gravitational inverse square law has been experimentally tested, which are currently probing the sub-millimetre range. For $d$ between 2 and 6, the size of the extra dimensions varies from a millimetre to a fermi. The relation between the 4-dimensional Planck scale $M_P$ and the scale $M_S$ in $(4+d)$-dimension is

$$M_P \approx M_S^{(d+2)} R^d \,, \tag{1}$$

where $R$ is the compactification radius. In the ADD model, because $R$ is large it is possible to lower $M_S$ down to a TeV and avoid the hierarchy problem. An important consequence of the lowering of the Planck scale is that quantum gravity effects could be discernible at energies of $\mathcal{O}$(TeV) and, consequently, a whole new class of studies of the effects of gravitons at present and future colliders becomes possible.

The graviton propagating in the full $4+d$ dimensions, manifests in 4-dimensions as a tower of massive Kaluza-Klein (KK) modes. These modes interact with the SM particles confined to the 3-brane via the energy-momentum tensor. Each KK mode couples to the SM particles with a coupling of the order of $1/M_P$. Even though the coupling of each KK mode to the SM particles are highly suppressed. The effective coupling is obtained after summing over all the KK modes and due to the high multiplicity of the KK modes the effective interaction has a strength of $1/M_S$ [2, 3]. This enhanced interaction strength provides viable signatures of the graviton KK modes at colliders. Both real graviton production and the effects of virtual gravitons in various processes have been studied in the literature [4] and have yielded bounds on $M_S$ which are in the ball-park of a TeV.

Existing collider studies of the effects of gravitons have been done at the Born level in QCD. It is important to compute next-to-leading order QCD (NLO-QCD) to these processes in order to quantify the size of the QCD correction and to see how robust the leading order estimate of the cross-section is with respect to the QCD correction. Clearly this impacts the experimental studies of the graviton-mediated processes (and graviton production processes) crucially. It is with this motivation that the present paper presents the results of the computation of NLO-QCD corrections to $e^+e^- \to$ hadrons via $\gamma$, Z and graviton exchange. These results are used to study the impact of the QCD correction for this process at the proposed linear $e^+e^-$ collider which is planned to operate at centre-of-mass energies between 500 GeV and 1.2 TeV. The results we present here are for the ADD model, but since the QCD corrections that we present here are model-independent they may equally be used for studying the Randall-Sundrum model of warped compactification [5].



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We work with the following action:

$$\mathcal{S} = \mathcal{S}_{SM} - \frac{1}{2}\kappa \int d^4x \; \Theta^{QCD}_{\mu\nu}(x) \; h^{\mu\nu}(x)\,, \tag{2}$$

where $S_{SM}$ is the Standard Model action, $h^{\mu\nu}$ is the graviton field and $\kappa$ is the strength of the interaction.

The energy momentum tensor in QCD is given by

$$\begin{aligned}
\Theta^{QCD}_{\mu\nu} &= -g_{\mu\nu}\mathcal{L}_{QCD} - F^a_{\mu\rho}F^{a\rho}_{\nu} - \frac{1}{\xi}g_{\mu\nu}\partial^\rho(A^a_\rho\partial^\sigma A^a_\sigma) + \frac{1}{\xi}(A^a_\nu\partial_\mu(\partial^\sigma A^a_\sigma) + A^a_\mu\partial_\nu(\partial^\sigma A^a_\sigma)) + \\
&\quad + \frac{i}{4}\Big[\overline{\psi}\gamma_\mu(\overrightarrow{\partial}_\nu - igT^aA^a_\nu)\psi - \overline{\psi}(\overleftarrow{\partial}_\nu + igT^aA^a_\nu)\gamma_\mu\psi + \overline{\psi}\gamma_\nu(\overrightarrow{\partial}_\mu - igT^aA^a_\mu)\psi - \\
&\qquad - \overline{\psi}(\overleftarrow{\partial}_\mu + igT^aA^a_\mu)\gamma_\nu\psi\Big] + \partial_\mu\overline{\omega}^a(\partial_\nu\omega^a - gf^{abc}A^c_\nu\omega^b) + \\
&\quad + \partial_\nu\overline{\omega}^a(\partial_\mu\omega^a - gf^{abc}A^c_\mu\omega^b)\,.
\end{aligned} \tag{3}$$

In the above equation, $\xi$ is the gauge fixing parameter. We work in the Feynman gauge in which the gauge parameter $\xi = 1$. We have displayed explicitly the ghost terms with the ghost fields $\omega^a(x)$ since they contribute to our one-loop virtual corrections to the process under study. The presence of the ghost fields introduces two new vertices viz:

1. graviton-ghost-ghost vertex,

$$\Gamma_{\mu\nu}(p_1, p_2) = -i\frac{\kappa}{2}\delta^{ab}C_{\mu\nu,\rho\sigma}\,p_1^\rho\;p_2^\sigma\,, \tag{4}$$

2. graviton-ghost-ghost-gluon vertex,

$$\Gamma_{\mu\nu,\rho}(p_1, p_2) = -\frac{\kappa}{2}gf^{abc}C_{\mu\nu,\rho\sigma}\;p_2^\sigma\,, \tag{5}$$

where

$$C_{\mu\nu,\rho\sigma} = g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\mu\nu}g_{\rho\sigma}\,,$$

the $\mu,\nu$ index refers to the graviton field and $p_1,\ b$ and $p_2,\ a$ are the momenta and colour index of the incoming and out going ghost field respectively. Other than the rules for these vertices, our Feynman rules are the same as that given in ref. [3].[1]

We start by considering $e^+\ e^-$ scattering to hadronic final states,

$$l^-(k_1) + l^+(k_2) \to X(P_X)\,, \tag{6}$$

where $l^-, l^+$ are the incoming leptons and $X$ are the final-state hadrons and $k_1, k_2$ and $P_X$ are their respective momenta. The cross-section can be factorised into a leptonic part $\mathcal{L}_i(k_1,k_2)$ and a hadronic part $\mathcal{W}_i(q)$, as follows:

$$\begin{aligned}
\sigma^{e^+\ e^-}(k_1,k_2) &= \frac{1}{2s}\sum_{i=\gamma,Z,G}\int\frac{d^nq}{(2\pi)^n}(2\pi)^n\delta^{(n)}(q-k_1-k_2)\,\times \\
&\quad \times\mathcal{L}_i(k_1,k_2)\cdot\mathcal{P}_i(q)\cdot\mathcal{P}_i(q)\cdot\;\mathcal{W}_i(q)\,,
\end{aligned} \tag{7}$$

[1] The only exceptions are the Feynman rules for the fermion-antifermion-gauge boson-graviton vertex and the three gauge boson-graviton vertex which differ from those of ref. [3] by an overall sign.

where $n$ is the space-time dimension, $q$ is the momentum of the intermediate photon, $Z$ or graviton and $\sqrt{s}$ is the center of mass energy,

$$\begin{aligned} s &= (k_1 + k_2)^2 \,, \\ &= q^2 \equiv Q^2 \,. \end{aligned} \tag{8}$$

The propagator $\mathcal{P}_G(q)$ is given by

$$\mathcal{P}_G(q) = i \; B_{\mu\nu\rho\sigma}(q)\mathcal{D}(q^2) \,, \tag{9}$$

where,

$$B_{\mu\nu\rho\sigma}(q) = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{3}\eta_{\mu\nu}\eta_{\rho\sigma} \,,$$

with

$$\eta_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2} \,,$$

The function $\mathcal{D}(q^2)$ results from the sum over the time-like propagators of the KK modes:

$$\mathcal{D}(q^2) = 16\pi \left( \frac{Q^{d-2}}{\kappa^2 M_S^{d+2}} \right) I\left(\frac{M_S}{Q}\right) , \tag{10}$$

the summation over the non-resonant KK modes yields

$$\begin{aligned} I(\omega) &= -\sum_{k=1}^{d/2-1} \frac{1}{2k}\omega^{2k} - \frac{1}{2}\log(\omega^2 - 1) & d = \text{even}, \\ &= -\sum_{k=1}^{(d-1)/2} \frac{1}{2k-1}\omega^{2k-1} + \frac{1}{2}\log\left(\frac{\omega+1}{\omega-1}\right) & d = \text{odd}\,, \end{aligned} \tag{11}$$

where $\omega = M_S/Q$.

The leptonic tensor involves just the computation of square of the matrix element for the process $e^+ + e^- \to \gamma^*/Z^*(q)$ and $e^+ + e^- \to G^*(q)$ and is given as

$$\mathcal{L}_i(k_1, k_2) = \frac{1}{4}\sum_{spin} |M^{e^+ \, e^- \to i}|^2 \qquad i = \gamma, Z, G \,. \tag{12}$$

The hadronic part $\mathcal{W}_i(q)$ is computed using

$$\begin{aligned} \mathcal{W}_i(q) = &\int \prod_{j=1}^{m} \left( \frac{d^n p_j}{(2\pi)^n} (2\pi)\delta^+(p_j^2) \right) \times \\ &\times (2\pi)^n \delta^n \Big( q + \sum_{j=1}^{m} p_j \Big) |M^{i \to \sum_{j=1}^m p_j}|^2 \qquad i = \gamma, Z, G \,. \end{aligned} \tag{13}$$

In the following, we compute leading (LO) and next-to-leading order(NLO) contributions to the hadronic part within perturbative QCD in powers of the strong-coupling constant $\alpha_s$. Since we have gravity-mediated process, in addition to the usual SM process, we have two classes of diagrams. The first class contains only photon or $Z$ in the

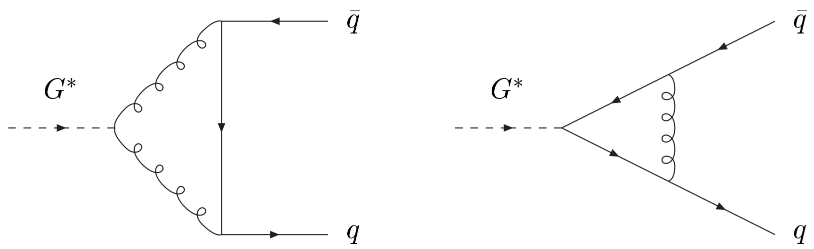


**Figure 1:** Graviton-quark-antiquark vertex at one loop.

intermediate state and the second class contains only gravitons. The interference between SM and gravity channels is identically zero in the fully inclusive reaction. Notice that the interference term will always be proportional to the product of a third-rank leptonic tensor and a third-rank hadronic tensor contracted via the propagator tensors of either $\gamma$ or $Z$ with that of the graviton. For the photon-graviton case, the inclusive hadronic tensor can be written only in terms of $g_{\mu\nu}$ and $q_\rho$ (the Levi-Cevita tensor will not appear in this). We know that there is no third rank tensor, say $S^{\mu\nu\rho}$ which can be constructed out of $g_{\mu\nu}$ and $q_\rho$ $(q.q \neq 0)$ that satisfies $q_\mu S^{\mu\nu\rho} = 0$ (as it should be for the theory where we have gravity coupled to a conserved energy momentum tensor). In the case of Z-graviton interference, the hadronic tensor can be proportional to a tensor, say, $S^{\mu\nu\rho} = \epsilon^{\mu\nu\rho\sigma} q_\sigma$ which satisfies $q_\mu S^{\mu\nu\rho} = 0$, but this when contracted with the Graviton propagator vanishes identically. Hence, there is no graviton-SM interference contribution to inclusive s-channel cross section (one can also argue out based on angular momentum conservation in field theories).

The leading-order contribution to the hadronic part comes from

$$\gamma^*/Z^* \rightarrow q + \bar{q}\,, \tag{14}$$

and at next to leading order level we have

$$\begin{aligned} \gamma^*/Z^* &\rightarrow q + \bar{q} + \text{one loop}\,, \\ \gamma^*/Z^* &\rightarrow q + \bar{q} + g\,. \end{aligned} \tag{15}$$

When the graviton is exchanged, we have two tree-level contributions

$$\begin{aligned} G^* &\rightarrow q + \bar{q}\,, \\ G^* &\rightarrow g + g\,. \end{aligned} \tag{16}$$

At NLO, the following processes, as shown in figure 1–4, contribute

$$\begin{aligned} G^* &\rightarrow q + \bar{q} + \text{one loop}\,, \\ G^* &\rightarrow q + \bar{q} + g\,, \\ G^* &\rightarrow g + g + \text{one loop}\,, \\ \\ G^* &\rightarrow g + g + g\,. \end{aligned} \tag{17}$$

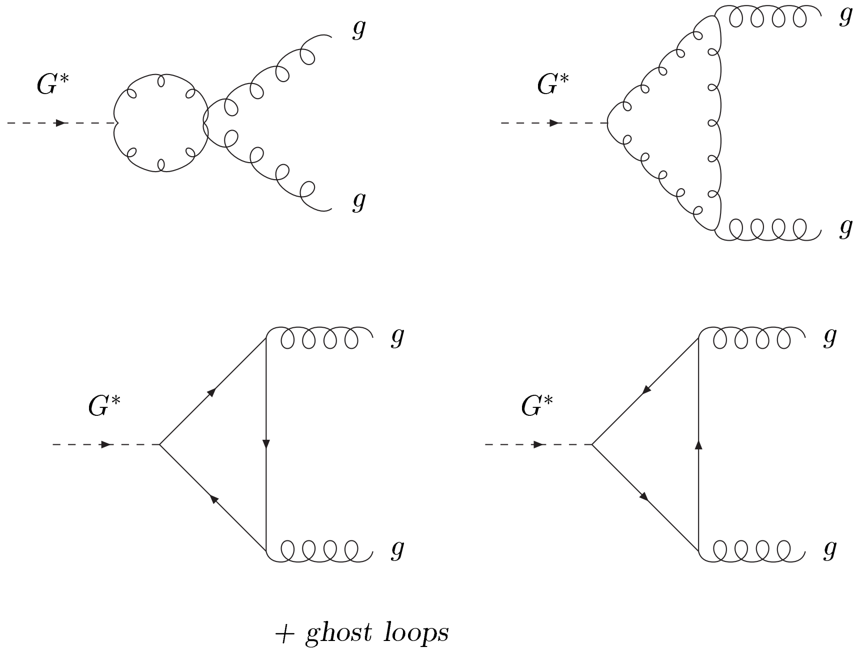


**Figure 2:** Graviton-gluon-gluon vertex at one loop.

The next-to-leading order corrections involve the computation of one-loop virtual gluon corrections and real gluon bremsstrahlung contributions to leading-order processes. Since we are dealing with the energy-momentum tensor coupled to gravity expressed in terms of renormalised fields and massless quarks, there is no overall ultraviolet renormalisation required. In other words, the operator renormalisation constant for the energy-momentum operator is identical to unity to all orders in perturbation theory. But we encounter infrared divergences (soft and collinear) in our computation. We have used dimensional regularisation to regulate these divergences. To do this, we have defined $n = 4 + \varepsilon$ where $n$ is space-time dimension. With this, all these divergences appear as $1/\varepsilon^\alpha$ where $\alpha = 1, 2$. Since the gravitons couple directly to quarks and gluons, there are many one-loop diagrams which appear to this order. But most of them do not contribute when quarks are taken to be massless. The contributing diagrams are given in the figures 1 and 2. Since throughout our calculation we have used the Feynman gauge, the gravitons also couple to ghost fields which can appear in loops. The real gluon-emission diagrams are given in the figures 3 and 4. In the figure 4, three identical gluons in the final state are permuted with the appropriate statistical factor. The soft divergences coming from virtual gluons and bremsstrahlung contributions cancel exactly as expected. Similarly, the remaining collinear divergences disappear in the fully inclusive reaction.

The hadronic part can be expanded in powers of

$$a_s = \frac{\alpha_s}{4\pi}\,, \tag{18}$$

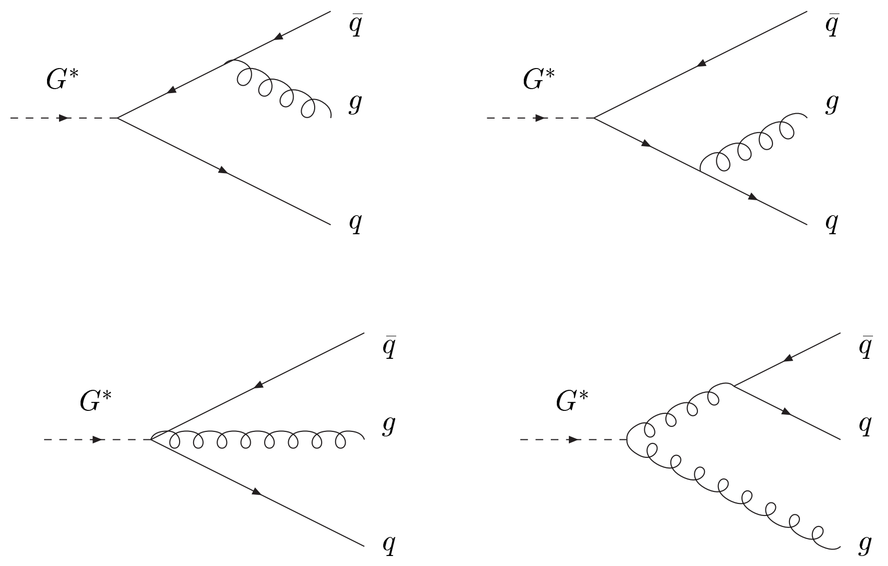


**Figure 3:** $G^* \to q + \bar{q} + g$.

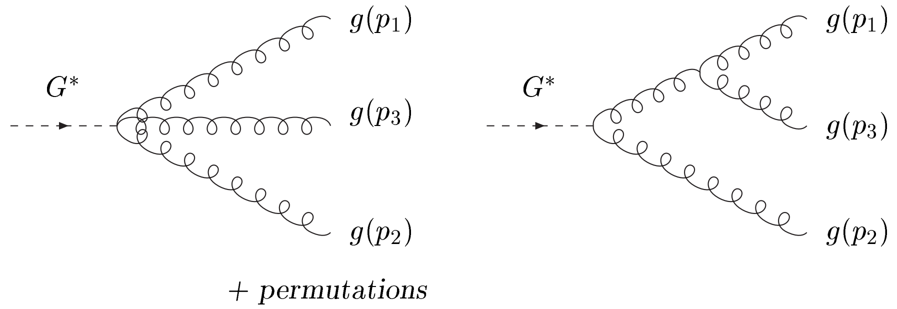


**Figure 4:** $G^* \to g + g + g$.



as follows:

$$\mathcal{W}_i = \mathcal{W}_i^{(0)} + a_s\, \mathcal{W}_i^{(1)} \qquad i = \gamma, Z, G\,. \tag{19}$$

After adding all the diagrams and folding in the appropriate colour factors we arrive at a simple-looking result for $\mathcal{W}_i^{(0)}$ and $\mathcal{W}_i^{(1)}$ for the non-SM part

$$\begin{aligned}
\mathcal{W}^{(0)}_{G,\mu\nu\rho\sigma} &= \frac{Q^4\kappa^2}{\pi}\ \left[(N^2-1)\left(\frac{1}{320}\right) + N\ n_f\left(\frac{1}{640}\right)\right] B_{\mu\nu\rho\sigma}(q)\,,\\
\mathcal{W}^{(1)}_{G,\mu\nu\rho\sigma} &= a_s\ \frac{Q^4\kappa^2}{\pi}\ \left[(N^2-1)C_A\left(-\frac{1}{144}\right) + N n_f C_F\left(\frac{7}{1152}\right)\right] B_{\mu\nu\rho\sigma}(q)\,.
\end{aligned} \tag{20}$$

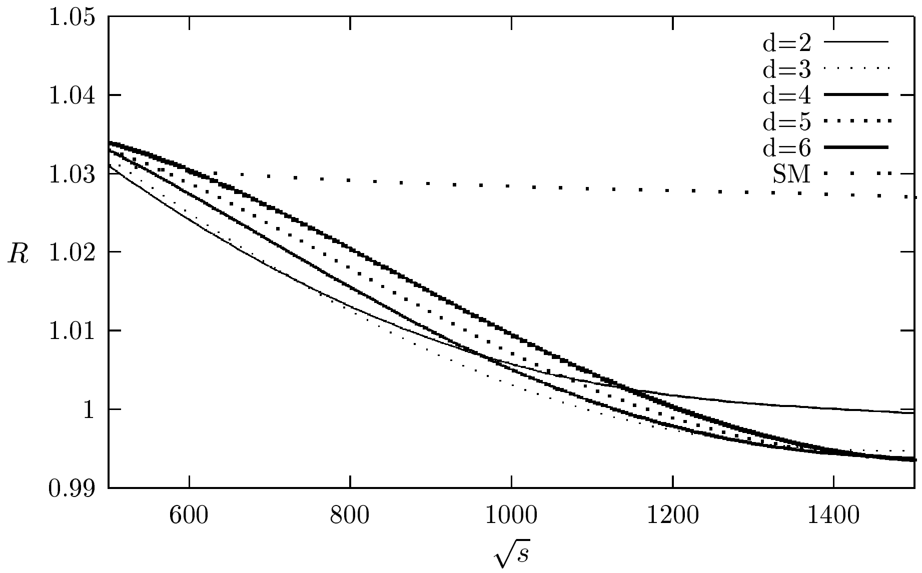


**Figure 5:** The ratio $R$ of the LO+NLO cross-section to the LO cross-section as a function of the $e^+e^-$ centre-of-mass energy, for number of extra dimensions, $d$, between 2 and 6. The SM reference curve is also shown.

The colour factors appearing in the above equations are

$$C_F = \frac{N^2-1}{2N}, \qquad C_A = N, \tag{21}$$

for SU($N$) gauge theory and $n_f$ is the number of flavours. The total cross section is found to be

$$\begin{aligned} \sigma^{SM} &= \sum_q \frac{4\pi\ \alpha^2}{3\ s} N \left\{ Q_q^2 - Q_q\ \frac{2s}{c_w^2 s_w^2}\ g_e^V\ g_q^V \frac{(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} + \frac{s^2}{c_w^4 s_w^4}({g_e^V}^2 + {g_e^A}^2) \times \right. \\ &\qquad \left. \times\ ({g_q^V}^2 + {g_q^A}^2)\frac{1}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \right\} \Big(1 + a_s\ (3C_F)\Big), \\ \sigma^G &= \frac{\kappa^4}{2560\pi s} Q^8\ \left[\mathcal{D}(q^2)\right]^2 \left\{ (N^2-1)\left(1 + a_s\ \left(-\frac{20}{9}\right) C_A\right) + \right. \\ &\qquad \left. + \ \frac{N n_f}{2}\left(1 + a_s\ \frac{35}{9} C_F\right) \right\}, \end{aligned} \tag{22}$$

where

$$\begin{aligned} c_w &= \cos\theta_W\,, & s_w &= \sin\theta_W\,, \\ g_a^V &= \frac{1}{2}T_a^3 - s_w^2 Q_a\,, & g_a^A &= -\frac{1}{2}T_a^3\,, \end{aligned} \tag{23}$$

and $Q_a$ is the electric charge of quarks and leptons.

We now proceed to estimate the magnitude of the QCD corrections presented above for the process $e^+e^- \to$ hadrons at the Linear Collider. In figure 5), we have plotted the ratio, $R$ as a function of the linear collider centre-of-mass energy, where

$$R = \frac{\sigma_{\mathrm{LO+NLO}}}{\sigma_{\mathrm{LO}}}\,. \tag{24}$$

The figure shows $R$ as a function of $\sqrt{s}$ for different values of the number of extra dimensions, $d$. Also shown in the figure is the ratio $R$ for the SM case. For our estimates, we have used $\alpha_s$ evaluated at the scale $s$ and for the running of $\alpha_s$ we have used $n_f = 5$ and $\Lambda = 0.215$ GeV. While the corrections are typically of the order of a couple of percent, they could still be significant because of the large statistics for this process at the Linear Collider (even if one assumes a conservative $50 - 100\,\mathrm{fb}^{-1}$ luminosity). Because of the the gluon-graviton couplings, the corrections coming from the graviton-exchange diagrams give a negative contribution to the NLO corrections, which tends to drive $R$ to values below 1 at large centre-of-mass energies.

In conclusion, we have presented the first computation of NLO-QCD corrections to graviton-mediated processes in the context of models of large extra dimensions. The process that we have considered is $e^+e^- \to$ hadrons via $\gamma$, $Z$, and graviton exchange. We have discussed the impact of the QCD corrections for the study of the ADD model using this process at a Linear Collider.



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## References

[1] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315]; *Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity*, *Phys. Rev.* **D 59** (1999) 086004 [hep-ph/9807344]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398];

[2] G.F. Giudice, R. Rattazzi and J.D. Wells, *Quantum gravity and extra dimensions at high-energy colliders*, *Nucl. Phys.* **B 544** (1999) 3 [hep-ph/9811291].

[3] T. Han, J.D. Lykken and R.J. Zhang, *On Kaluza-Klein states from large extra dimensions*, *Phys. Rev.* **D 59** (1999) 105006 [hep-ph/9811350].

[4] For an exhaustive list of references on the subject, see: A. Perez-Lorenzana, *Theories in more than four dimensions*, *AIP Conf. Proc.* **562** (2001) 53–85 [hep-ph/0008333]; K. Cheung, *Collider phenomenology for models of extra dimensions*, hep-ph/0305003; early papers on the collider phenomenology of the ADD model are: E.A. Mirabelli, M. Perelstein and M.E. Peskin, *Collider signatures of new large space dimensions*, *Phys. Rev. Lett.* **82** (1999) 2236 [hep-ph/9811337]; J.L. Hewett, *Indirect collider signals for extra dimensions*, *Phys. Rev. Lett.* **82** (1999) 4765 [hep-ph/9811356]; K.M. Cheung and W.Y. Keung, *Direct signals of low scale gravity at $e^+e^-$ colliders*, *Phys. Rev.* **D 60** (1999) 112003 [hep-ph/9903294];

P. Mathews, S. Raychaudhuri and K. Sridhar, *Getting to the top with extra dimensions*, *Phys. Lett.* **B 450** (1999) 343 [hep-ph/9811501]; *Large extra dimensions and deep-inelastic scattering at HERA*, *Phys. Lett.* **B 455** (1999) 115 [hep-ph/9812486]; *Testing TeV scale quantum gravity using dijet production at the Tevatron*, *J. High Energy Phys.* **07** (2000) 008 [hep-ph/9904232];
K. Agashe and N.G. Deshpande, *Limits on low scale gravity from* $e^+e^- \to w^+w^-, zz$ *and* $\gamma\gamma$, *Phys. Lett.* **B 456** (1999) 60 [hep-ph/9902263].

[5] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].